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Perfect Cuboid Problem

Vimal Kumar
Assistant Professor,
Dept. of Mathematics,
Govt. Degree College,
Babrala-Gunnaur, Sambhal,
Uttar Pradesh, India

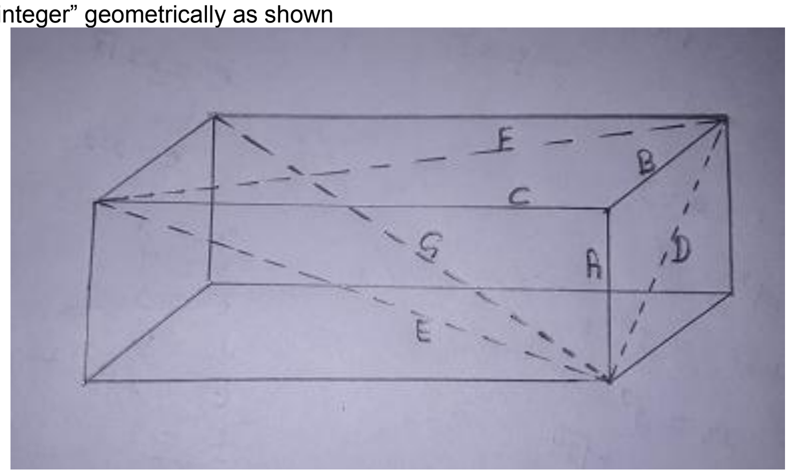
Abstract

In this paper i am trying to solve the Perfect Cuboid problem by taking different values of edeges which gives some integer value for face diagonals but internal diagonal remains non integer, taking consideration from the analysis for the integer and non integer values we are able to find the behaviour of Internal (space) diagonal with the help of unique factorization theorem.

Keywords: Perfect Cubiod, Integers, Face Diagonal, Internal Diagonal, Prime Numbers, Brick, Edges, Pythagorean Theorem, Unique Factorization Theorem.

Introduction

In this paper we discuss Perfect Cuboid problem which states that “the sum of square of three side edges is equal to the square of the internal diagonal (space diagonal) of the cuboid, where all the values remain integer” geometrically as shown



In this picture the three egdes are of length A,B,and C E,F are face diagonal where G is the Internal(Space)diagonal.the goal is to find a cuboid in which $A^2 + B^2 + C^2 = G^2$ where all the numbers A,B,C,D,E,F,G are integers.

This is the oldest unsolved problem in number theory and all in mathematics this problem is also called brick problem, perfect box problem ,Rational cuboid problem or A primitive Euler brick problem whose edge lengths are relatively prime in geometry.the search for such cuboid is still on, despite of grand researchs on finding such cuboid.

Aim of the Study

The aim of study of this topic is to check the hypothesis about Perfect Cuboid and to improve the mathematical point to create the interest for learners in geometry (mathematics) .

Review of Literature

Euler give the idea to solve this problem in which by taking different values of edges as A,B,C he got the face diagonal value given by the equation

$$A^2 + B^2 = D^2$$
$$A^2 + C^2 = \mathrm{E}^2$$
$$B^2 + C^2 = F^2$$

According to Euler only three edges and face diagonal values are integers but space diagonal is non integer. As different values taken by Paul halcke in 1719 for solving brick problem.

A	B	C	D $A^2 + B^2 = D^2$	E $A^2 + C^2 = E^2$	F $B^2 + C^2 = F^2$
44	117	240	125	244	267
85	132	720	157	715	732
140	480	693	500	707	843
160	231	792	281	808	825
187	1020	1584	1037	1595	1884
195	748	6336	773	6339	6380
240	252	275	348	365	373
429	880	2340	979	2379	2500

For all values in above table face diagonals are integers but internal diagonal are non integers.

Analysis

Consider all the edges has the same non zero value let say

$$let A = 2, B = 2, C = 2, D = 2\sqrt{2}, E = 2\sqrt{2}, F = 2\sqrt{2}, G = 2\sqrt{3}$$

In this case we get all face diagonal and internal (space) diagonal as non integer.

Now if any two of the edges are have the same value and third is different value say

$$let\ A = 3, B = 3, C = 4\ then\ D = 3\sqrt{2}, E = 5, F = 5, G = \sqrt{34}$$

In this case we find two face diagonal are integer and one face diagonal and internal diagonal are non integers.

As different value taken by Paul halcke in above table gives all face diagonal as integer value but internal (space) diagonal is non integers. From the above anaylsis problem is partially solved , in order to solve the problem completely , we have to prove either the internal diagonal is integer or non integer (i.e not all the seven values A,B,C,D,E,F,G are integers).

The proof in support that the internal diagonal is non integer as follows in first row of the table where

$$A = 44, B = 117, C = 240, D = 125, E = 244, F = 267, G = \sqrt{73225} = 270.601$$

Here G is non integer, " since we know that square root of a positive integer is either an integer or irrational" if the resulting number is a square number then square root of resulting number is rational then problem is solved. But it can not be found by taking different value of edges.

Now the resulting number is not a square number now 'the problem is to show that square root of that non square resulting number is irrational"

Let the square root of the resulting non square number is rational number

$$\sqrt{n} = \frac{p}{q} \text{ where } p, q \in N, q \neq 0, p \neq 0$$

$$\text{then } n = \frac{p^2}{q^2}$$

If q=1, then, $\sqrt{n}\,(\text{resulting number}) = P$ which shows that n is a square number

So $q \neq 1$ since $\sqrt{n} > 1$ then $p > q > 1$

By the unique factorization theorem of integers which states that "every integer greater than 1 can be expressed as product of its primes, therefore p is product its primes. And in p^2 there will be even number of primes. Similarly q can be expressed as product of its primes and there will be even number of primes in q^2 , we can also express n as a product of primes.since n (resulting number) is not a square number then there exists at least one prime number that has an odd number of primes therefore there exists at least one prime in product of nq^2 that has odd number of primes since $nq^2 = p^2$ which is contradiction to the result that a number can neither be even and odd both

hence the resulting number $=\sqrt{n}$ is irrational.

Conclusion

As from the above analysis if we take the edges of same length we not getting all face diagonal and internal diagonal as integer but in some examples as where A=3,B=4,C=4 then we can get two face diagonal as integer one face diagonal and integral diagonal is non integer ,as discussed by Paul halcke in number of examples for integer edges which gives face diagonal as integer value but internal diagonal is not an integer with the help of unique factorization theorem it is concluded that the internal diagonal is non integer.

References

Buter, B "Durango bill's The "Integer Brick Problem (The Euler Brick Problem)".

Elements of Number Theore I. M. Vinogradov, ISBN-048-678-165-8

Shaffer Sherrill (1987). "Necessary Divisors of perfect Integer Cuboids".Abstracts of the American mathematical society.

Kraitchik, M. (1945). "On certain rational cuboids". Scripta mathematica."

Roberts Tim (2010). "Some constraints on the existence of a perfect cuboid .Australian mathematics Society ISSN 1326-2297.

Weisstein, Ericw. "Perfect Cuboid" From Mathworld- A Wolfram Web Resource.